BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D.C. 20024

B70 06049

SUBJECT: Estimate of Daily Fuel Required to Control Gravity Gradient and Solar Precession during an Artificial G Experiment - Case 620

DATE: June 15, 1970

FROM: R. J. Ravera
W. W. Hough

ABSTRACT

An artificial gravity experiment is one of several options being considered for the proposed second Skylab. If the solar panel configuration of the first Skylab is maintained, it is desirable that the Z-axis be the spin axis and that it be parallel to the solar vector. Rotational stability, however, requires that the spin axis be the axis of maximum moment of inertia. The vehicle Z-axis and axis of maximum moment of inertia can be approximately aligned by modifying the Skylab I configuration.

Bias gravity gradient torque and apparent solar precession tend to cause the spin axis to rotate away from the solar vector. However, the spin and sun vectors can be held aligned with the CSM reaction control system. The amount of RCS fuel required each day is presented as a function of the direction of spin and the sun line-orbit plane angle, β. The spin direction can be chosen to minimize fuel requirements as the β angle history during the experiment will be known beforehand.

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MEMORANDUM FOR FILE

Introduction

An artificial gravity mission option is being considered for the proposed second Skylab. The artificial g field is to be obtained by spinning the vehicle about its mass center. Assuming that the planar solar arrays are, as in the first Skylab, perpendicular to the vehicle Z-axis, it is then desirable that the vehicle Z-axis be the spin axis and that it be aligned parallel to the solar vector (see Figure 1). However, the vehicle Z-axis is not necessarily the axis of maximum moment of inertia (3-axis), and rotational stability requires that the spin axis be the 3-axis. With the addition of ballasting beams to the Skylab I configuration, the 3-axis can be brought to within about 5° of the vehicle Z-axis.(1) Other Skylab II configurations under study (without the ATM) can, with careful attention to mass distribution, achieve similar alignment.

It is perfectly feasible to initially align the spin angular momentum vector with the 3-axis and with the solar vector, and the power penalty is almost negligible. But there are two effects which tend to move the 3-axis off the sun line. These are the bias gravity gradient torque which causes a precession of the spin angular momentum vector and the apparent solar precession. This memorandum computes the daily CSM RCS fuel required to compensate for these precession effects.

Gravity Gradient Torque

Before discussing the effect of gravity gradient torque, it is beneficial to introduce the spin angular momentum vector*, $\underline{H}$, defined by

$$\underline{H} = I_3 \underline{\omega}_3 \tag{1}$$

*Underlined letters represent vector quantities; the letter alone represents the magnitude of the vector.

where I_3 is the maximum principal (3-axis) moment of inertia and $\underline{\omega}_3$ is the stable spin angular velocity vector.

The bias gravity gradient torque vector, $\underline{T}_{GG}$, acts in the orbital plane and tends to cause primarily a precession of the spin angular momentum vector. Results of more thorough analyses, now in progress, support this contention. This effect is pictured in Figure 2. The change in angular momentum due to $\underline{T}_{GG}$, $\Delta\underline{H}_{GG}$, is perpendicular to the sun line and is parallel to the orbital plane. The magnitude of $\underline{T}_{GG}$ averaged over an orbit is

$$(T_{GG})_{AVG} = \frac{3}{8}\,\omega_0^{\,2}\,(2I_3 - I_1 - I_2)\,\sin 2\beta \tag{2}$$

where ω_0 is the orbital rate, I_1, I_2, and I_3 are the moments of inertia about the vehicle minimum, intermediate and maximum principal axes of inertia, respectively, and β is the minimum angle between the orbital plane and the sun line. On a per orbit basis,

$$\Delta H_{GG}\ (\text{per orbit}) = \int_0^T (T_{GG})_{AVG}\,dt = (T_{GG})_{AVG}\,T \tag{3}$$

where T is the orbital period. It is clear from (2) and (3) that ΔH_{GG} is dependent on the sign of β. For the conditions of Figure 2, with $\underline{H}$ toward the sun, $\Delta\underline{H}_{GG}$ is directed out of the page for $\beta > 0$ and into the page for $\beta < 0$.

Solar Precession

Apparent solar precession is the effect due to the earth's rotation about the sun; its rate, $\dot{\gamma}$, assumed constant, is

$$\dot{\gamma} = \frac{360°}{364.25\ \text{days}} = 0.9856\ °/\text{day}.$$

It appears as though the vehicle is experiencing, with respect to the sun, a change in angular momentum, $\dot{H}_{sp}$, where

$$\dot{H}_{sp} = H\dot{\gamma}$$

or from (1),

$$\Delta H_{sp} \text{ (per orbit)} = I_3 \omega_3 \dot{\gamma} T. \tag{4}$$

The vector $\Delta \underline{H}_{sp}$ is perpendicular to the plane formed by the sun line and the normal to the ecliptic. For the conditions of Figure 2, $\Delta \underline{H}_{sp}$ is directed out of the page.

Combined Solar Precession - Gravity Gradient Precession Control

Over a 30-day mission, $\Delta \underline{H}_{GG}$ changes direction (inclination and sense) with respect to $\Delta \underline{H}_{sp}$. It then follows from the relationship

$$\underbrace{|\Delta \underline{H}_{GG} + \Delta \underline{H}_{sp}|}_{\text{COMBINED ESTIMATE}} \quad < \quad \underbrace{|\Delta \underline{H}_{GG}| + |\Delta \underline{H}_{sp}|}_{\text{SEPARATE ESTIMATE}}$$

that the best estimate of required fuel is obtained by compensating for the combined effect of solar and gravity gradient precession. Direct addition of the vector magnitudes would result in an unrealistically conservative fuel requirement. In order to estimate the needed fuel, a knowledge of the included angle, ϕ, between $\Delta \underline{H}_{GG}$ and $\Delta \underline{H}_{sp}$ is required. The angle ϕ is dependent on launch parameters, orbit parameters and time of year. However, the extreme values of ϕ as a function of β can be computed and the upper and lower bounds on the required fuel follow directly. The Appendix has details of the derivation of the angle ϕ.

Fuel Estimates

Estimates of the daily fuel required to control gravity gradient and solar precession are presented in Figure 3. Fuel required (lbs/day) is plotted as a function of β angle. The following data formed the basis for the results;

h (orbit altitude)	=	235 NM
I_1	=	929,354 SLUG FT^2
I_2	=	4,404,947 SLUG FT^2
I_3	=	4,572,216 SLUG FT^2
I_{sp} (specific impulse of CSM RCS fuel)	=	273 SEC
i (orbit inclination)	=	50°
ω_3	=	0.6283 RAD/SEC = 6 RPM

The above inertia properties are derived from Skylab I, augmented for a one year mission, and include ballasting beams for mass properties control.

In addition to the limits of the angle ϕ, the fuel estimates account for the 27.75° and 62.25° off-sets of the CSM RCS thruster pods from the Skylab Y-axis. The distance from the CSM thrusters to the inertial control axis varies sinusoidily over the spin cycle. It was assumed that the thrusters would be fired over one-tenth of the spin cycle (see Figure 4) such that at the mid-point of the firing, the maximum available moment arm (41.75 feet) exists. The moment arm used for fuel estimates (39.01 feet) is the average that exists over the one-tenth circumference.

The bounds on the fuel estimates derive from two cases:

Case 1) The fuel calculation is based on the angle ϕ where ϕ is computed, from Eq. (A-7), using the maximum value of the angle between the ecliptic and orbital planes.

Case 2) The fuel calculation is based on the angle ϕ where ϕ is computed from Eq. (A-7) using the minimum value of the angle between the ecliptic and orbital planes for $-26.55° \leq \beta \leq 26.55°$ when $\Delta \underline{H}_{GG}$ and $\Delta \underline{H}_{sp}$ cannot be colinear. The remainder of the curve, $|\beta| > 26.55°$, is based on colinearity of the $\Delta \underline{H}_{GG}$ and $\Delta \underline{H}_{sp}$ vectors, in the same direction when β is positive and in opposite directions when β is negative.

The bounds merge at $\beta = \pm 73.45°$ since $\Delta \underline{H}_{GG}$ and $\Delta \underline{H}_{sp}$ are colinear at these points; they also merge at $\beta = 0$ since, from (1) and (2), $\Delta \underline{H}_{GG}$ is zero.

Conclusion

In this memorandum, the direction of vehicle spin was chosen so that $\underline{H}$ points toward the sun. From a fuel standpoint, it is clear from Figure 3 that the greatest economy can be achieved by conducting the experiment at a time when $\beta < +10°$. If the vehicle is spun in the opposite direction, so that $\underline{H}$ points away from the sun, then the sign of β on Figure 3 is reversed and the greatest economy results when $\beta > -10°$. Clearly, the fuel requirement for control of gravity gradient and solar precession can be minimized by selecting the direction of the spin vector, $\underline{\omega}_3$, so that solar precession opposes gravity gradient precession for as great a portion of the experiment as possible.

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Attachments
- Figures 1-4
- Appendix
- References

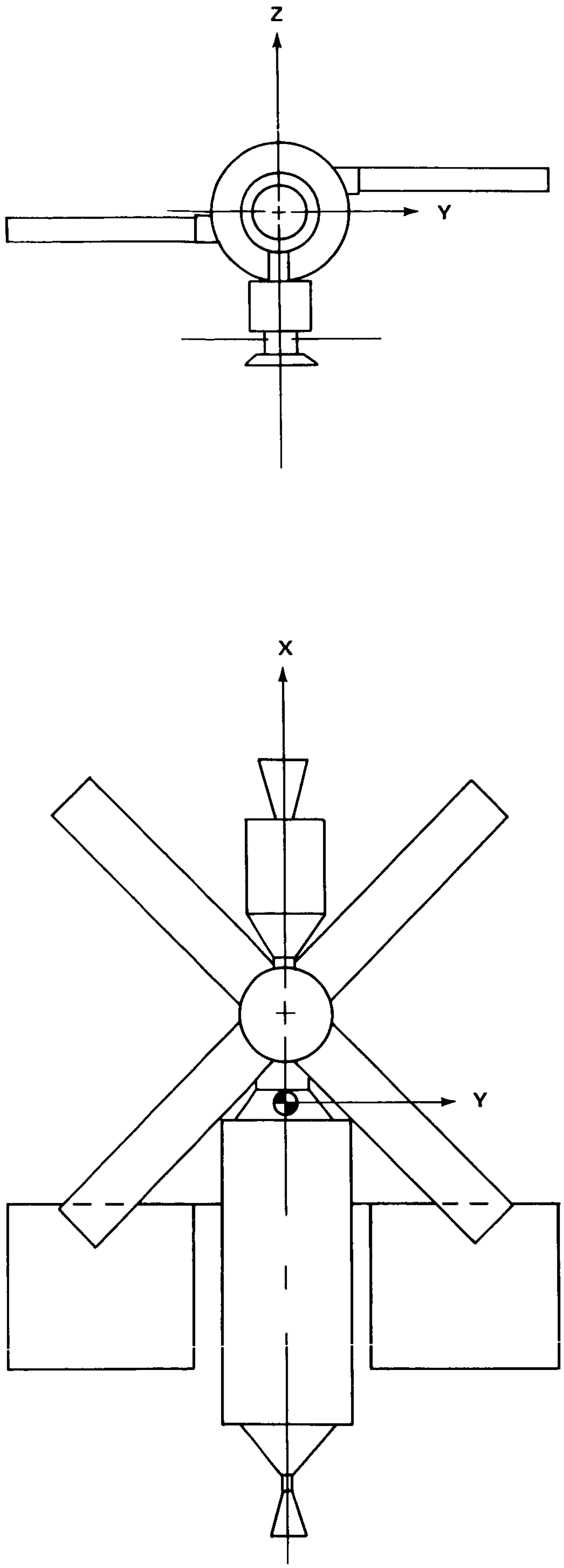
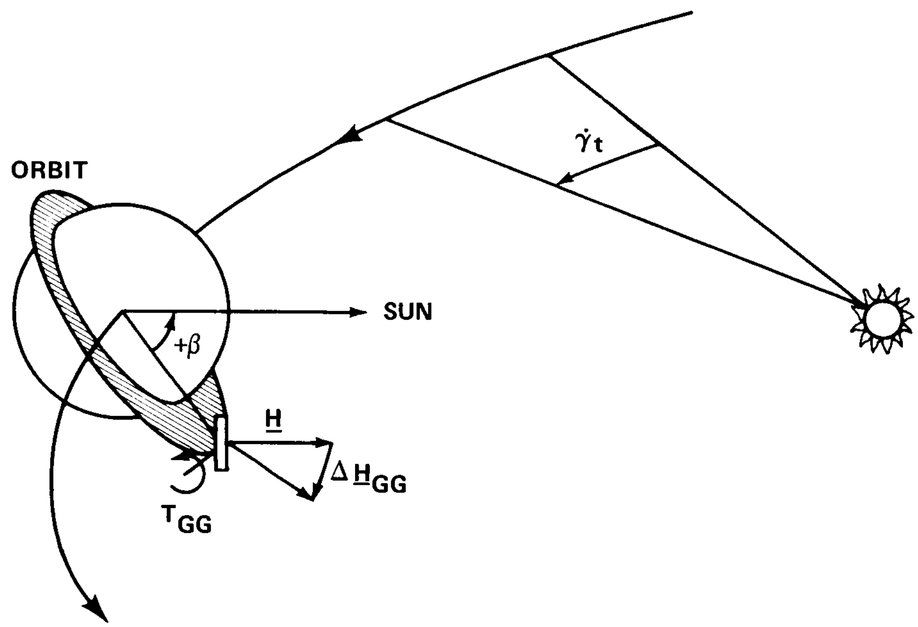


FIGURE 1 – SKYLAB CONFIGURATION



EFFECT OF GRAVITY GRADIENT TORQUE

FIGURE 2

BOUNDS ON DAILY FUEL REQUIRED VS. β TO KEEP MOMENTUM VECTOR ($\underline{H}$) ALONG SUN LINE
6 RPM

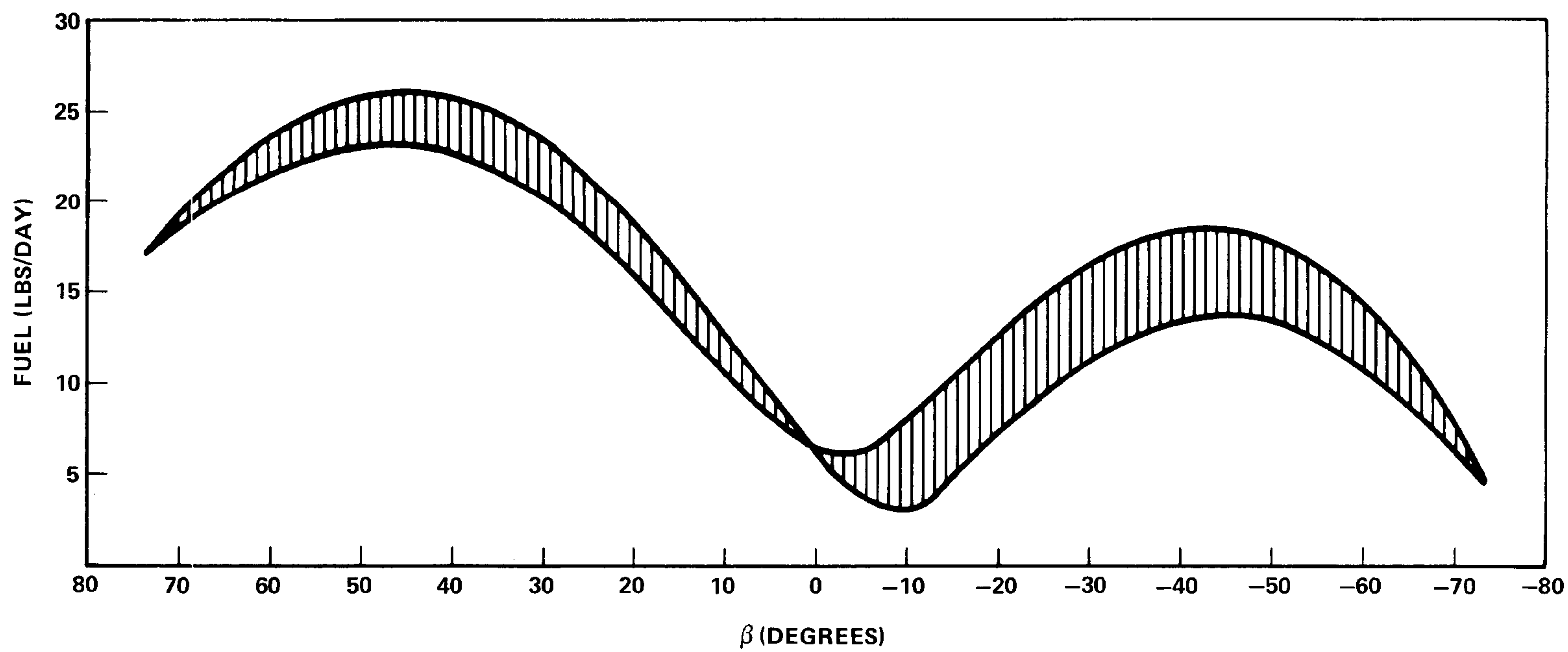


FIGURE 3

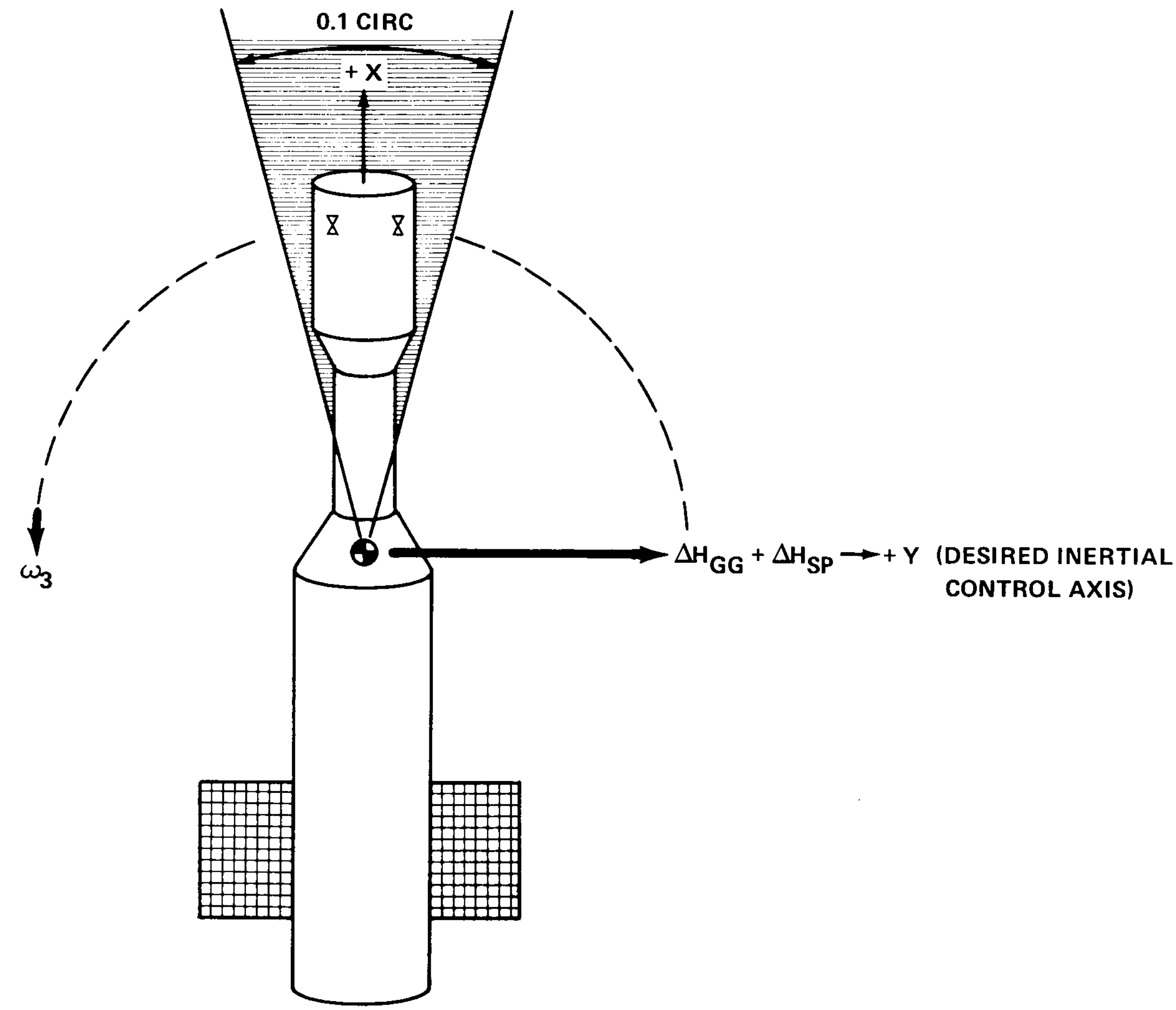


FIGURE 4 – POSITION OF VEHICLE AT THRUSTER FIRING

APPENDIX

The purpose of this Appendix is to compute the limits on ϕ, the included angle between $\Delta\underline{H}_{GG}$ and $\Delta\underline{H}_{sp}$, as a function of β. Figure A-1 illustrates the unit vector normal to the orbital plane, $\underline{n}_{op}$, and a vector normal to the ecliptic plane, $\underline{n}_{e}$. Both are in the ecliptic-north hemisphere, and the acute angle between them is ε. This angle ε varies with time because of orbital regression, but it is bounded between a minimum value

$$\varepsilon_{MIN} = i - e$$

and a maximum value

$$\varepsilon_{MAX} = i + e$$

where i is the orbital inclination and e is the angle between the ecliptic and equatorial planes (e = 23.45°). These limiting values of ε are related to the limiting values of ϕ. In Figure A-1, $\underline{S}$ is the solar vector. When ε is a fixed value (such as an above limit), $\underline{n}_{op}$ will lie on the illustrated cone, and its position on that cone determines β, the angle between the solar vector and the orbital plane (angle between $\underline{n}_{op}$ and the plane perpendicular to $\underline{S}$).

The unit vector $\underline{t}_{op}$ is perpendicular to $\underline{n}_{op}$ and $\underline{S}$ while the unit vector $\underline{t}_{e}$ is perpendicular to $\underline{n}_{e}$ and $\underline{S}$. The vector $\Delta\underline{H}_{GG}$ is directed along $\underline{t}_{op}$ and $\Delta\underline{H}_{sp}$ is directed along $\underline{t}_{e}$. ϕ is the angle between these vectors.

For a fixed ε, the tip of $\underline{n}_{op}$ can be anywhere along the rim of the cone in Figure A-1; when $\beta = 0$, $\phi = \varepsilon$ and when $\beta = \pm\varepsilon$, $\phi = 0$. The geometry of Figure A-1 yields the following relationships:

$$d = n_{op} \sin\varepsilon = \sin\varepsilon \tag{A-1}$$

and

$$\Delta = d \cos\theta = \sin\varepsilon \cos\theta \tag{A-2}$$

It can be observed that

$$d \sin\theta = n_{op} \sin\beta = \sin\beta \tag{A-3}$$

Combining (A-1) and (A-3)

$$\sin\theta = \frac{\sin\beta}{\sin\varepsilon} \tag{A-4}$$

The quantity Δ can also be expressed by

$$\Delta = n_{op} \cos\beta \sin\phi = \cos\beta \sin\phi \tag{A-5}$$

Equating (A-5) and (A-2) leads to

$$\cos\beta \sin\phi = \sin\varepsilon \cos\theta \tag{A-6}$$

Substituting for θ from (A-4), (A-6) becomes

$$\sin\phi = \frac{\sin\varepsilon}{\cos\beta} \cos\left[\sin^{-1}\left(\frac{\sin\beta}{\sin\varepsilon}\right)\right] \tag{A-7}$$

Equation (A-7) establishes the limits on ϕ, the angle between $\Delta\underline{H}_{GG}$ and $\Delta\underline{H}_{sp}$ when the cone angle, ε, takes on the previously mentioned values. ϕ is limited to the first quadrant, as $\Delta\underline{H}_{GG}$ can change sign by (2) and (3) of the text.

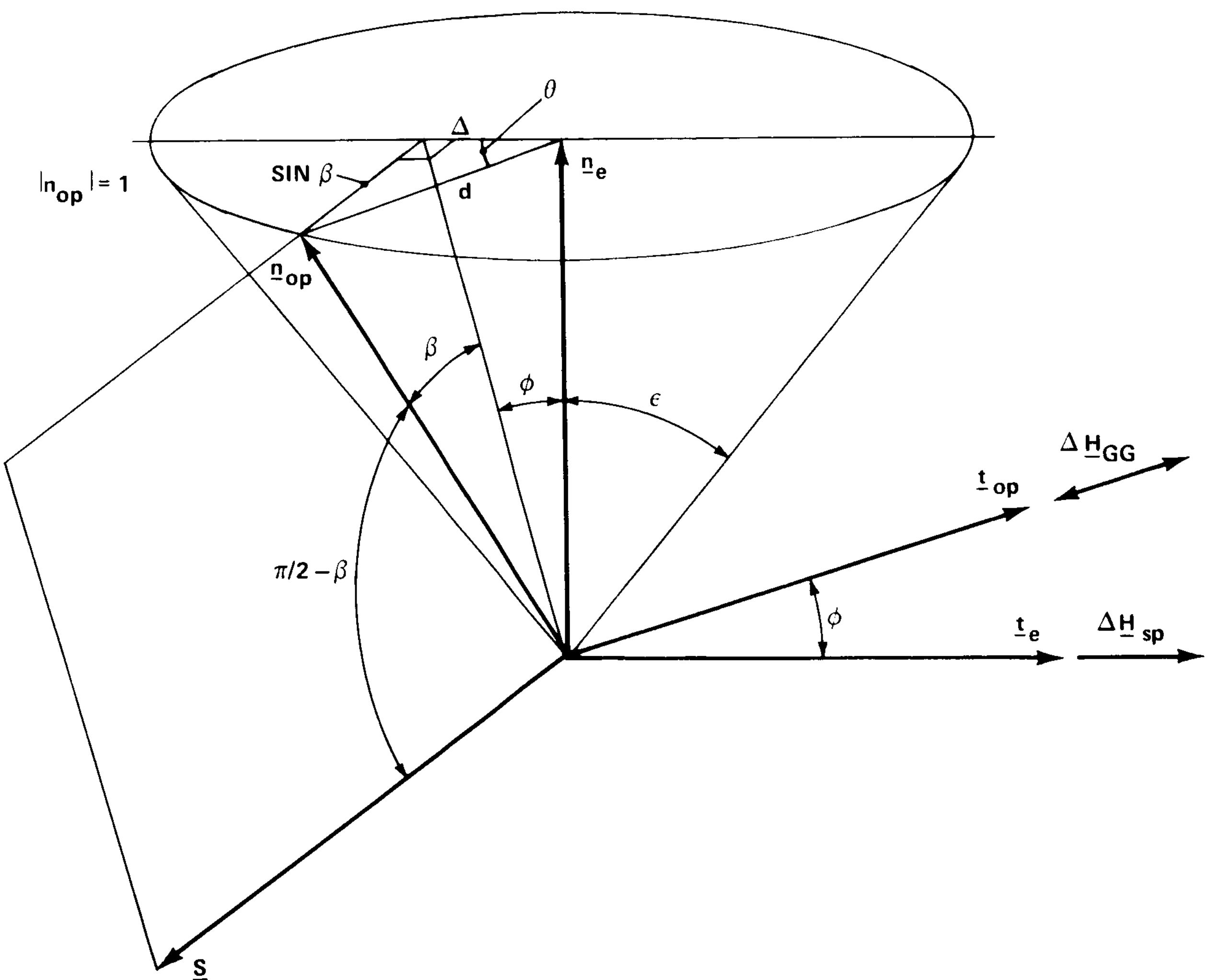


FIGURE A-1 - GEOMETRY FOR COMPUTATION OF ϕ

REFERENCES

1. Voelker, L. E., "Influence of Vehicle Dynamics on the Artificial Gravity Experiment on the Second Saturn Workshop," Bellcomm TM-70-1022-5, April 17, 1970.

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